NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3089

ONE-DIMENSIONAL, COMPRESSIBLE, VISCOUS FLOW RELATIONS APPLICABLE TO FLOW IN A DUCTED HELICOPTER BLADE

By John R. Henry

Langley Aeronautical Laboratory
Langley Field, Va.

Washington
December 1953

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SUMMARY

One-dimensional, steady-state, compressible, viscous flow relations are presented which permit the determination of flow conditions at any radial position in a ducted helicopter blade. The relations are required for estimating the performance of proposed helicopter jet-propulsion systems which involve ducting air or gases through the blade from root to tip. A limited number of calculations over a wide range of helicopter operating conditions and relative duct sizes are also presented. The "choking" problem in the straight duct is discussed.

INTRODUCTION

The use of jet engines as propulsive units for helicopters is being investigated by many agencies. Several different power-plant configurations are under consideration, including those which require the ducting of air or gases through the length of the helicopter blade. Analytical relations are presented herein for evaluating the conditions of the ducted flow at any point of its travel from the blade root to the tip. Such information is required in order to determine limiting quantities of flow, optimum duct sizes, overall losses, and flow properties (i.e., static pressure, stagnation pressure, and Mach number) of the gases delivered to an engine unit at the blade tip.

In subsonic ducted flow the action of wall friction and turbulence losses in producing decreasing density and increasing Mach number is well known and has been described adequately in the literature (e.g., ref. 1). The limit to this action is the attainment of a Mach number of unity or the "choking" condition. The choking problem may require careful consideration in the design of ducted helicopter blades because the passage length relative to the passage diameter necessarily will be large and the entering Mach number may be in the high subsonic range.

The effect on the ducted flow of the centrifugal compression due to the blade rotation will be to raise the density and lower the Mach number, and thus counteract the effect of friction. The net effect of the two opposed actions will depend on the particular combination of flow conditions, duct geometry, and helicopter operating conditions and must be evaluated for each combination.

The analysis presented herein is limited to steady-state, one-dimensional, compressible, viscous flow and applies only to straight ducting.

SYMBOLS

A	duct cross-sectional area, ft^2
a	speed of sound, fps
B	rotation parameter, $M_R^2\left(\frac{D}{R}\right)^2 \frac{T_0}{T_t}$
c_p	specific heat at constant pressure, Btu/lb/$^{\circ}$F
c_v	specific heat at constant volume, Btu/lb/$^{\circ}$F
D	duct hydraulic diameter, $\frac{4A}{\text{Perimeter}}$
f	friction factor from reference 2
g	acceleration due to gravity, ft/sec^2
J	Joule's constant, 778 ft-lb/Btu
M	duct-flow Mach number
M_R	rotor tip Mach number, $\Omega R/a_0$
p_t	duct-flow stagnation pressure, lb/ft^2 abs
p	duct-flow static pressure, lb/ft^2 abs
r	radial distance from center of hub to blade element, ft
R	blade radius, ft
N_{Re}	Reynolds number of ducted flow at station 1, $\rho_1 V_1 D_1/\mu_1$

T static temperature, $^{\circ}R$

T_t duct-flow stagnation temperature, $^{\circ}R$

V duct-flow velocity, fps

x ratio of blade-element radius to rotor-blade radius, r/R

γ ratio of specific heats, c_p/c_v

ρ mass density, lb-sec^2/ft^4

Ω rotor angular velocity, radians/sec

μ coefficient of viscosity, lb-sec/ft^2

$\psi_a(M)$ Mach number function for heat term

$\psi_b(M)$ Mach number function for friction term

$\psi_c(M)$ Mach number function for centrifugal-force term

$\psi_d(M)$ Mach number function for area-change term

Subscripts:

0 atmospheric

1 duct station at hub center

2 duct station at rotor tip

x any radial station

ANALYSIS

For steady-state, one-dimensional, compressible, viscous, ducted flow subjected to a centrifugal force in the flow direction, as in flow through the length of a helicopter blade, the rate of change of momentum of a mass segment of the flow may be set equal to the algebraic sum of the forces on the segment (Newton's second law of motion) as follows:

$$\rho VA\ dV = -A\ dp - 4A \frac{f}{D} \frac{\rho V}{2} V\ dr + \Omega^2 \rho Ar\ dr - p\ dA \qquad (1)$$

Equation (1) assumes a constant molecular weight and specific heat, no drag-producing bodies in the stream, and no changes in mass flow rate. On the right-hand side of the equation, the first term represents the pressure force, the second, the friction force, the third, the centrifugal force due to the duct or blade rotation, and the fourth, the pressure force due to a change in area.

Equation (1) may be converted into terms of stagnation temperature and Mach number through use of the following: the continuity relation; the perfect-gas law; and the expression relating stagnation temperature, static temperature, and Mach number.

The converted equation is as follows:

$$\frac{dM}{dr} = \frac{M\left(1 + \gamma M^2\right)\left(1 + \frac{\gamma - 1}{2} M^2\right)}{2T_t\left(1 - M^2\right)} \frac{dT_t}{dr} + \frac{2\gamma f M^3\left(1 + \frac{\gamma - 1}{2} M^2\right)}{D\left(1 - M^2\right)} -$$

$$\frac{M\left(1 + \frac{\gamma - 1}{2} M^2\right)^2 \Omega^2 r}{g\left(c_p - c_v\right) J T_t\left(1 - M^2\right)} - \frac{M\left(1 + \frac{\gamma - 1}{2} M^2\right)}{1 - M^2} \frac{\frac{dA}{A}}{dr} \qquad (2)$$

A study of reference 3 will show that equation (2) is equivalent to equation [19] of that reference except for terms which the previously enumerated assumptions eliminate and the centrifugal-force term which was not considered in reference 3.

Equation (2) may be nondimensionalized and expressed in terms of a rotation parameter B as follows:

$$\frac{dM}{d\frac{r}{D}} = \psi_a(M) \frac{dT_t}{T_t\ d\frac{r}{D}} + \psi_b(M)f - \psi_c(M)B \frac{r}{D} - \psi_d(M) \frac{\frac{dA}{A}}{d\frac{r}{D}} \qquad (3a)$$

where

$$\psi_a(M) = \frac{M\left(1 + \gamma M^2\right)\left(1 + \frac{\gamma - 1}{2} M^2\right)}{2\left(1 - M^2\right)} \tag{3b}$$

$$\psi_b(M) = \frac{2\gamma M^3\left(1 + \frac{\gamma - 1}{2} M^2\right)}{1 - M^2} \tag{3c}$$

$$\psi_c(M) = \frac{\gamma M\left(1 + \frac{\gamma - 1}{2} M^2\right)^2}{1 - M^2} \tag{3d}$$

$$B = M_R^{\,2}\left(\frac{D}{R}\right)^2\left(\frac{T_0}{T_t}\right) \tag{3e}$$

and

$$\psi_d(M) = \frac{\psi_b(M)}{2\gamma M^2} \tag{3f}$$

The solution of equation (3a) requires the use of additional relations such as expressions for the duct-flow stagnation temperature and the duct cross-sectional area in terms of radial position in the duct r or the differential equations thereof. Such an expression for the stagnation temperature would permit the evaluation of the first term on the right-hand side of equation (3a), which may be classified as a heat term. The exact nature of such an expression depends on many design details and on flow and operating conditions, and its detailed analysis is beyond the scope of this paper. A few general remarks, however, are in order.

The influence of the heat term would be to increase the Mach number if heat were added and to decrease the Mach number if heat were transferred from the ducted flow. Heat would be added to the flow if the blade duct were used as a combustion chamber, in which case the relation of stagnation temperature to duct length would be a function principally of the space rate of combustion and associated parameters. A certain amount of heat transfer from the flow will generally occur with or without combustion because the ducted-flow temperature will be higher than the local stagnation-temperature recovery on the external blade surface if it is assumed that the ducted flow is compressed mechanically before it enters the blade duct. At any station in the duct the amount of heat transfer, and thus the temperature of the ducted flow, would depend on the heat-transfer coefficient, the stagnation-temperature recovery of the ducted flow, and the stagnation-temperature recovery on the external surface of the blade. The heat-transfer coefficient would be determined to a large extent by the blade and blade-duct design.

The second term on the right-hand side of equation (3a) accounts for wall friction effects and would tend to increase the Mach number. The friction factor f is a function of Reynolds number and the degree of roughness of the duct surface. In the Reynolds number range of 10^6 to 10^7, the data of reference 2 indicate that the following empirical relations apply:

For commercial pipes (steel, cast iron),

$$f = \frac{0.0247}{N_{Re}^{0.141}}$$

For smooth surface (glass, copper, drawn tubing),

$$f = \frac{0.0236}{N_{Re}^{0.153}}$$

These relations indicate that at a given Reynolds number the friction factor for rough surfaces is about 30 to 40 percent greater than for smooth surfaces. Increasing the Reynolds number from 10^6 to 10^7 produces about the same percentage decrease in the friction factor. It is shown in a subsequent section that changes of these magnitudes in friction-factor values do not affect the solution of equation (3a) appreciably.

The third term of equation (3a) accounts for the influence of the blade rotation, which is to compress the gas and reduce the Mach number. The term contains a rotation parameter B, defined by equation (3e), which involves the rotor tip Mach number, the ratio of duct hydraulic diameter to blade radius, and the ratio of atmospheric temperature to the duct-flow stagnation temperature. It will be noted that the definition of rotor tip Mach number,

$$M_R = \frac{\Omega R}{a_0} \tag{4}$$

depends only on the tip speed due to rotation and the atmospheric speed of sound.

The fourth term in equation (3a) accounts for the effects on Mach number of changes in cross-sectional area and is determined by the duct geometry.

Equations (3a) to (3f) provide a means for obtaining the Mach number variation along the blade duct. The relations used in converting equation (1) to equation (2) may be combined to provide an expression for the static-pressure ratio across a length of duct:

$$\frac{p_1}{p_2} = \frac{M_2}{M_1}\frac{A_2}{A_1}\sqrt{\frac{1 + \frac{\gamma - 1}{2} M_2^2}{1 + \frac{\gamma - 1}{2} M_1^2}}\sqrt{\frac{T_{t_1}}{T_{t_2}}} \tag{5}$$

Equation (5) may be converted to an equation for stagnation-pressure ratio through use of the relation between static pressure, stagnation pressure, and Mach number to produce

$$\frac{p_{t_1}}{p_{t_2}} = \frac{p_1}{p_2}\left(\frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2}\right)^{\frac{\gamma}{\gamma-1}} = \frac{M_2}{M_1}\frac{A_2}{A_1}\left(\frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\sqrt{\frac{T_{t_1}}{T_{t_2}}} \tag{6}$$

It is of interest to note that for adiabatic flow in a constant-area duct not subjected to centrifugal forces, the first, third, and fourth terms on the right-hand side of equation (3a) are eliminated. The resulting equation can be integrated to give

$$\frac{M_x^2}{1 + \frac{\gamma - 1}{2} M_x^2}\, e^{\frac{2}{(\gamma+1)M_x^2}} = \frac{\dfrac{M_1^2}{1 + \frac{\gamma - 1}{2} M_1^2}\, e^{\frac{2}{(\gamma+1)M_1^2}}}{e^{\frac{2\gamma}{\gamma+1}4f\frac{r}{D}}} \tag{7}$$

Equation (7) represents an end condition for the helicopter blade-duct performance in which the rotational speed has been reduced to zero, and is of use in obtaining an overall view of the effect of the blade rotation.

NUMERICAL CALCULATIONS

In order to obtain a better understanding and a graphic view of the solution expressed in equation (3a), a few numerical evaluations were made which were directed toward investigating the choking problem. The calculations were limited for simplicity to the case of adiabatic flow in a constant-area duct. The limitation to adiabatic flow is justified on the basis that heat transfer from the flow would tend to decrease the Mach number and thereby prevent choking, so that the adiabatic-flow case is conservative in this respect, and heat addition in the blade duct (combustion) is not believed to be a strong possibility.

The calculations were based on assumed values of the rotation parameter B (see eq. (3e)) of 0, 0.0002, 0.0007, and 0.0014. The value of 0.0014 corresponds to a ratio of blade radius to duct diameter R/D of 26.7 for a rotor tip Mach number M_R of unity and a temperature ratio T_0/T_t of unity. These conditions are representative of an estimated maximum practical value of B or an estimated minimum practical value of R/D. A zero value of B corresponds to the other extreme of no rotational speed or infinite blade radius relative to the duct diameter. For adiabatic flow and no rotational speed, equation (7) is applicable.

For each assumed value of B, equation (3a) was evaluated for adiabatic flow for duct-inlet Mach numbers (at the hub center) of 0.3, 0.6, and 0.8 at an inlet Reynolds number of 10^7. The value of the friction factor f was taken from reference 2 and corresponded to commercial cast-iron pipe. The rough-surface friction factor was chosen in order to obtain a conservative answer relative to choking.

The adiabatic, constant-duct-area version of equation (3a) was evaluated numerically by a step-by-step integration process to obtain finite changes in Mach number for a series of values of $\Delta(r/D)$. The values of the Mach number functions of equations (3c) and (3d) were taken from precalculated curves presented in figure 1. A curve of equation (3b) has also been included in figure 1. In the Mach number range from 0.4 to 1.0 the reciprocal of the Mach number functions is presented to avoid having the ordinate approach infinity near a Mach number of 1.0.

The changes in Mach number in the blade duct were determined in increments of length equal to 4 duct diameters until a total duct length equal to 32 diameters was reached. The increment of length equal to

4 diameters was found to be small enough so that the Mach number could be calculated with an error less than 0.5 percent. In order to obtain these accuracies, however, it was necessary to base the calculation of the Mach number change across a given increment on the Mach number at the center of the increment as predicted from the change in Mach number across the preceding increment. After completing the calculation for a given increment it was necessary, for the first 8 diameters of duct length, to repeat the calculation on the basis of a corrected estimate of Mach number at the center of the increment.

RESULTS

The variation of duct-flow Mach number, as obtained by these procedures, is presented in figure 2. For the cases corresponding to the higher values of rotation parameter ($B = 0.0007$ and 0.0014) the centrifugal effects more than compensated for the friction effects, so that substantial decreases in Mach number occurred through the length of the duct. This result was obtained even at an inlet Mach number of 0.8, for which the stationary-blade curve ($B = 0$) indicates choking or a Mach number of 1.0 at a duct length of 7.8 diameters. For an inlet Mach number of 0.8, a value of B of 0.0002 produced a Mach number of 1.0 at a length of 10.3 diameters; however, a value of B of 0.0003 did not produce a choking condition and resulted in a net decrease in Mach number over the blade length.

The curves of figure 2 specify the Mach number variation in the blade duct for given values of the parameter B, inlet Mach number, and inlet Reynolds number. These inlet conditions, however, would actually be determined by compressor operating point, nozzle-exit area, and other factors. Therefore, in order to change the inlet conditions or move from one group of curves to another in figure 2, a change in one or more of these factors is implied.

The influence of Reynolds number or friction factor on the Mach number was investigated briefly. Calculations were made for $B = 0.0007$ at a Reynolds number of 10^6 in order to compare with results for the value of 10^7 of figure 2. Reducing the Reynolds number to 10^6 is equivalent to increasing the friction factor about 40 percent and resulted in a 3-percent increase in the Mach number. This relatively small influence of Reynolds number justifies ignoring small changes in Reynolds number in calculating the flow along the blade duct.

The Mach number at a duct length of 30 diameters as a function of inlet Mach number is presented in figure 3, which is a cross plot of figure 2. The effect of the rotation parameter B on the duct exit Mach number is clearly illustrated. An approximate choke line intersecting each B curve has been drawn in figure 3 by fairing through points corresponding to choking at some location in the duct. The line was faired through a point for $B = 0$ calculated from equation (7), through a point for $B = 0.0003$ determined by cross plots of figure 2, and asymptotic to the vertical at an inlet Mach number of 1.0, according to the reasoning that as the rotation parameter approaches large values the inlet Mach number must approach 1.0 in order to obtain choking. The choke line represents a locus of the minimum values of inlet Mach number for which a Mach number of 1.0 can be obtained at some location within the 30-diameter duct length. As the rotation parameter is increased from a value of zero, the radial location at which a Mach number of 1.0 will occur moves from the 30-diameter location upstream toward the hub, and the minimum inlet Mach number increases from a value of 0.657 and approaches 1.0. If the duct length were increased, the minimum inlet Mach number for choking when $B = 0$ would decrease according to equation (7).

The stagnation-pressure and static-pressure ratios over the 30-diameter length are presented in figure 4, which was obtained through use of equations (5) and (6) and the Mach number values of figure 3. The intercepts of the pressure-ratio curves on the ordinate axis are indicative of the pressure difference over the blade length due to centrifugal force with no internal air flow and were calculated from the expression

$$\frac{p_{t_2}}{p_{t_1}} = \frac{p_2}{p_1} = e^{\frac{\gamma B}{2(D/R)^2}}$$

which was obtained by integrating analytically between stations 1 and 2 the expression for the centrifugal force on an increment of gas mass. With values of rotation parameter B of 0.0007 and 0.0014, stagnation-pressure ratios of 1.55 and 2.41 were obtained with no flow, and the stagnation-pressure ratio decreased slightly with increasing inlet Mach number because of increasing friction losses. The increasing divergence of the curves for static-pressure ratio and stagnation-pressure ratio with increasing inlet Mach number is a result of the increasing difference between inlet and exit Mach numbers. (See fig. 3.)

CONCLUDING REMARKS

The differential equation presented for steady-state, one-dimensional, compressible, viscous flow was used in a step-by-step integration process to determine the Mach number changes for adiabatic flow in a ducted helicopter blade over a wide range of helicopter operating conditions and relative duct sizes. With adiabatic flow in a constant-area duct 30 diameters in length, a Mach number of 1.0 will not be attained for inlet Mach numbers of less than 0.657, which corresponds to the case of no blade rotation. As the duct length increases, this value of minimum inlet Mach number for choking will decrease. For a given duct length, as the rotation parameter increases, the minimum inlet Mach number for choking increases, and the choking point moves toward the inlet end of the duct.

The maximum pressure ratio across a length of duct occurs with no flow. The stagnation-pressure ratio decreases with increasing inlet Mach number because of increasing friction losses. For the range of rotation parameters assumed, a maximum stagnation-pressure ratio of 2.41 across a 30-diameter duct length was calculated for the adiabatic-flow case.

A change by a factor of 10 in the duct-flow Reynolds number, which determines the friction factor and friction losses, produced only a 3-percent change in the calculated duct-flow Mach number for the adiabatic-flow case with a moderate value of the blade rotation parameter.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., October 5, 1953.

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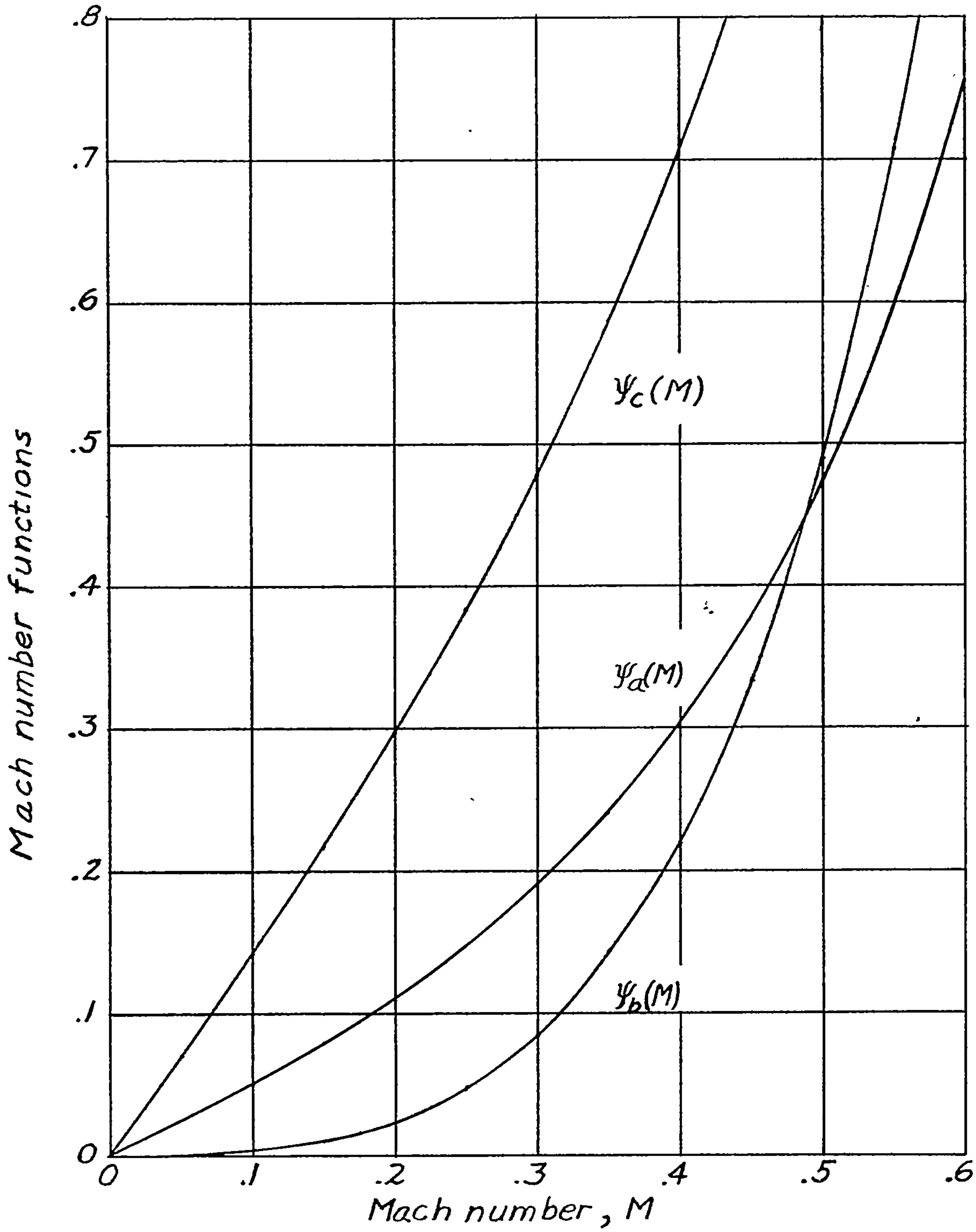
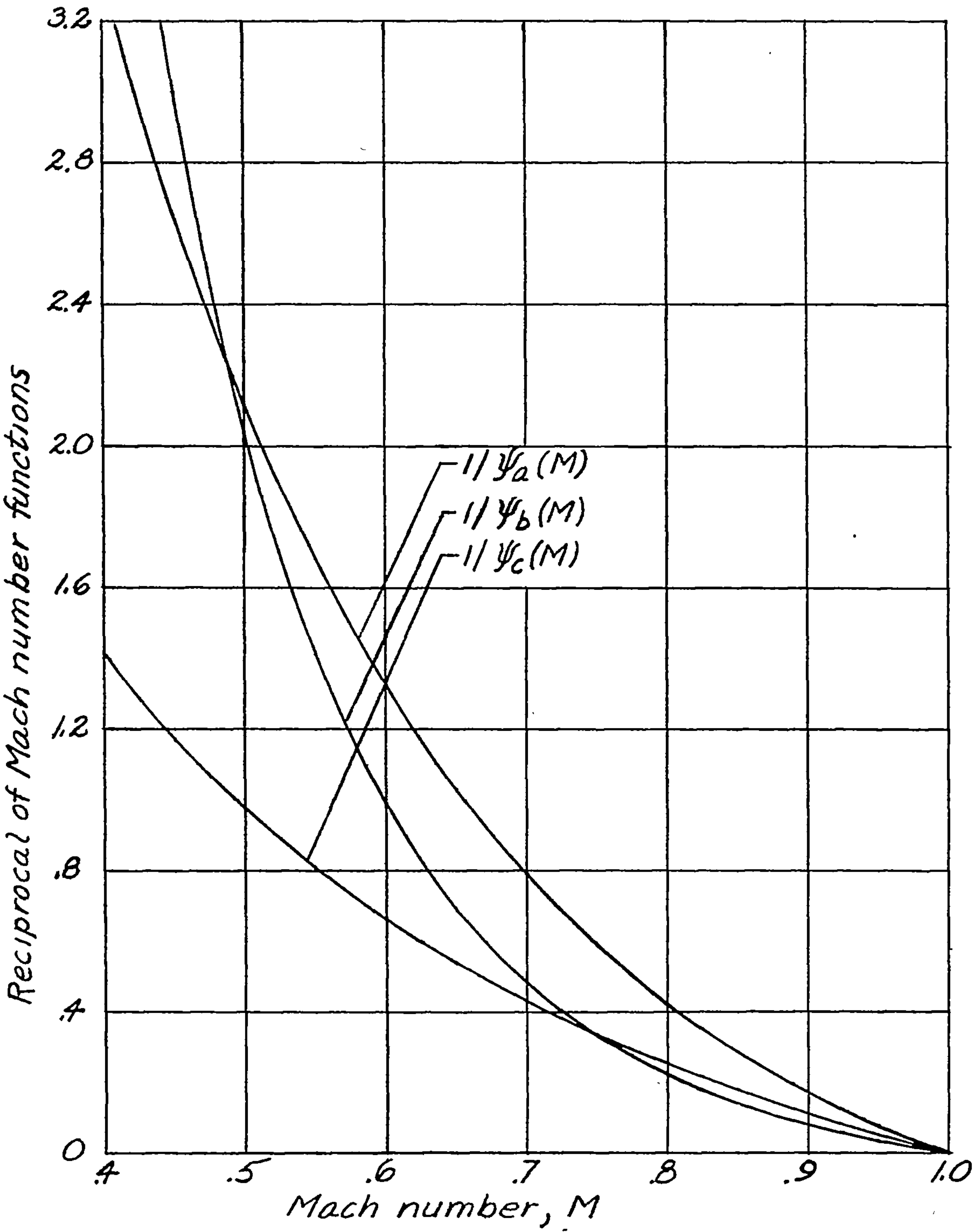


(a) M = 0 to 0.6.

Figure 1.- Mach number functions defined by equations (3b), (3c), and (3d).



(b) M = 0.4 to 1.0.

Figure 1.- Concluded.

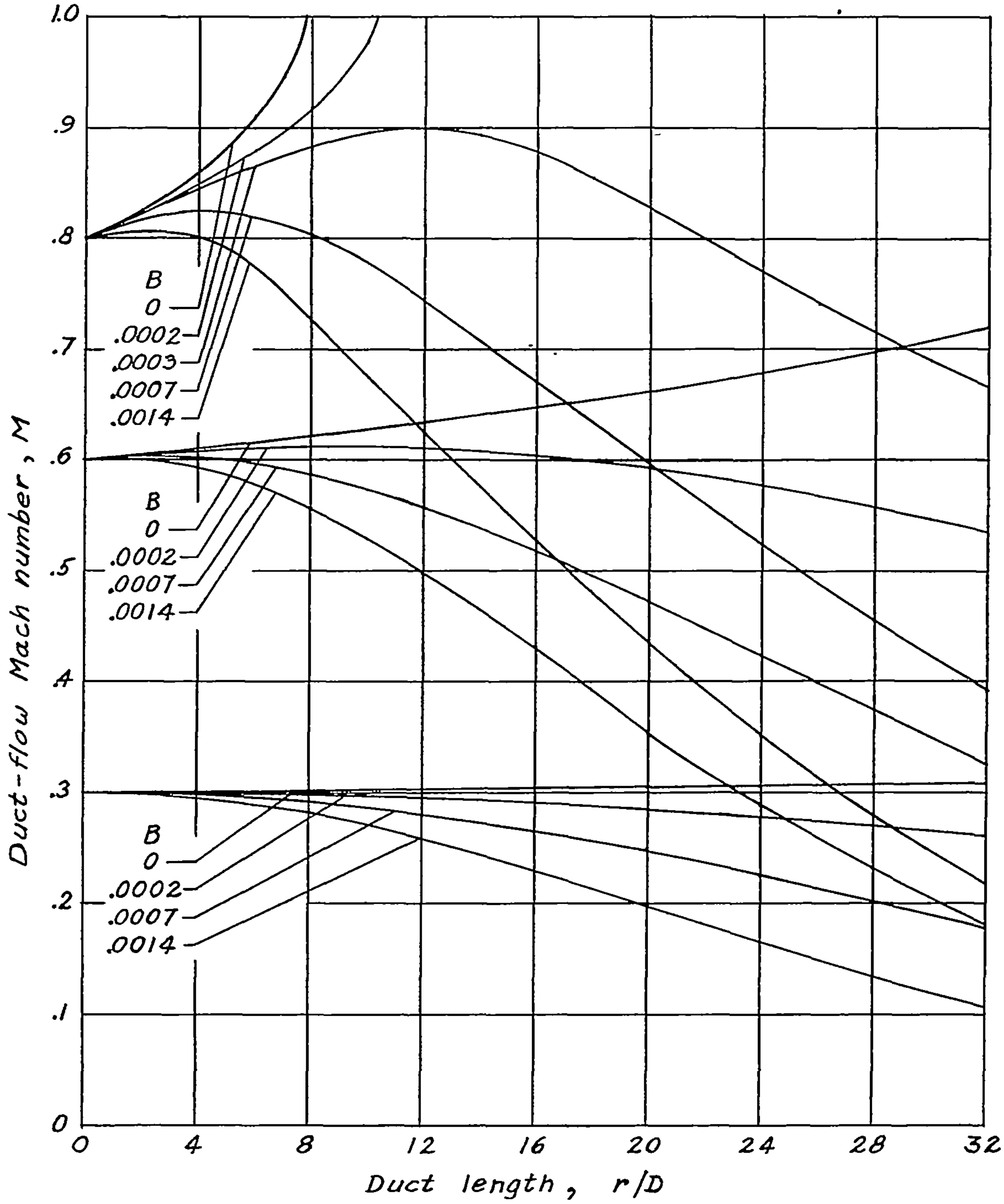


Figure 2.- Relation of duct-flow Mach number to duct length for constant values of $B = M_R^2(D/R)^2(T_0/T_t)$. $N_{Re} = 10^7$.

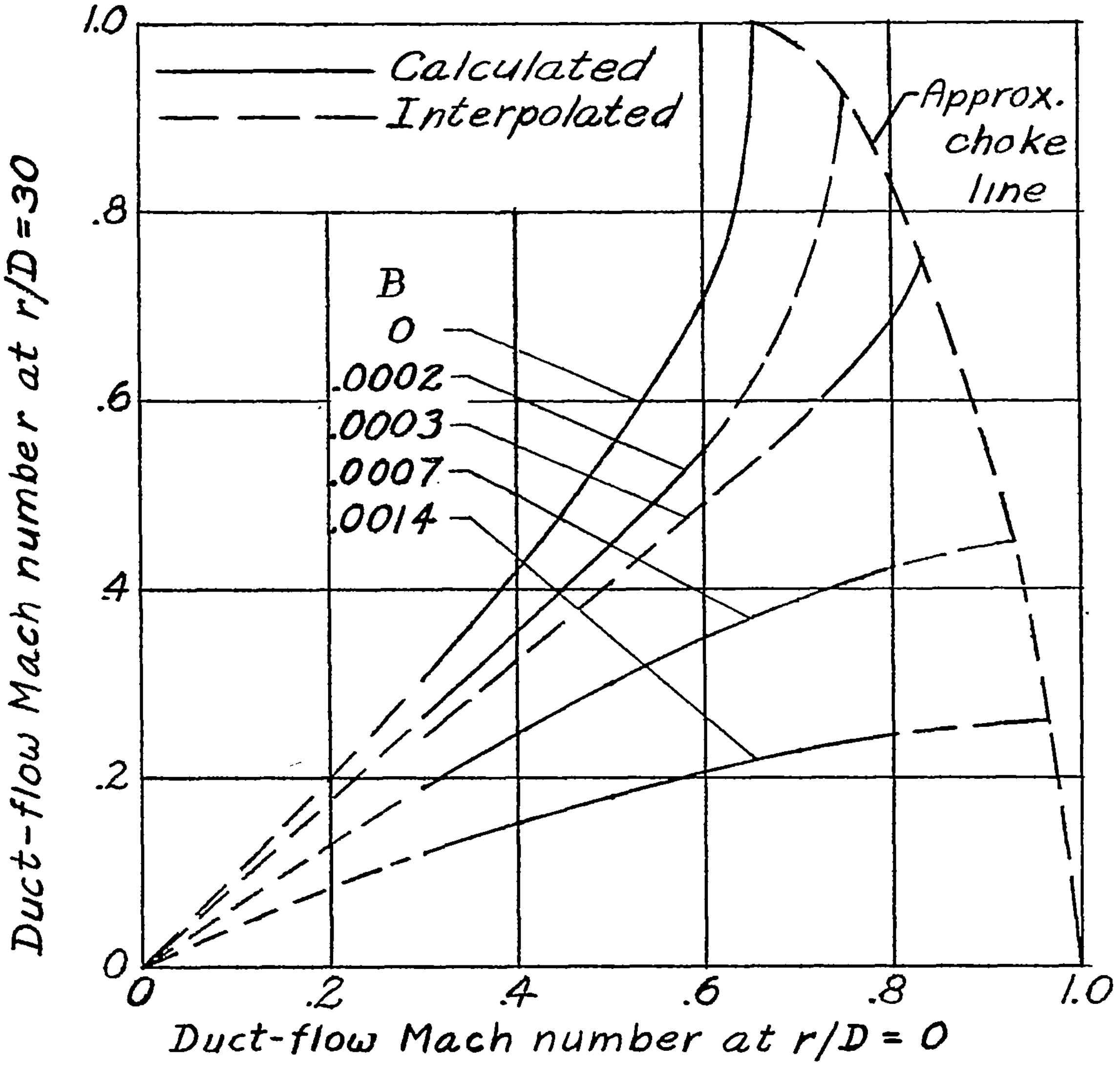


Figure 3.- Net Mach number change for duct length of 30 diameters.

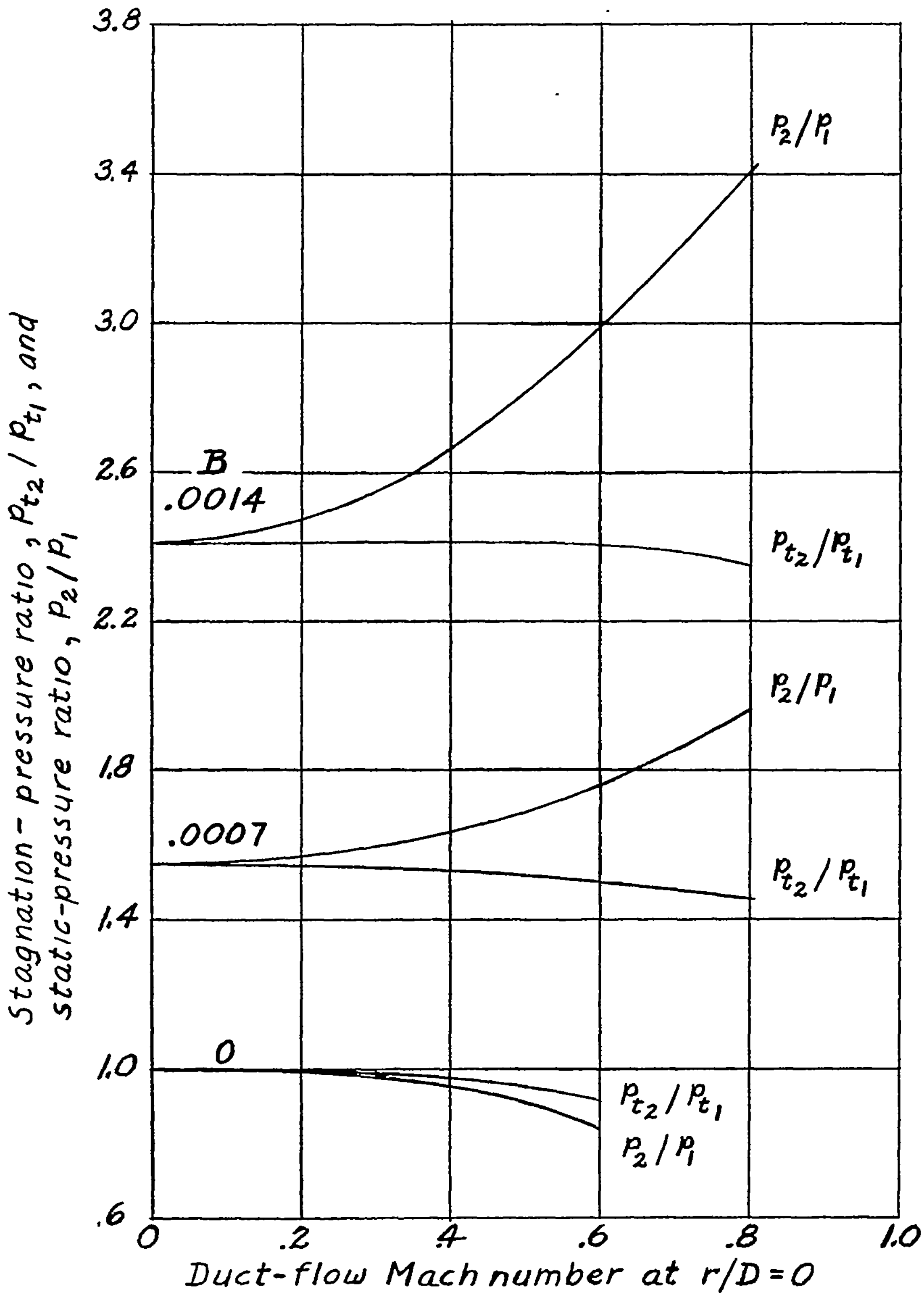


Figure 4.- Stagnation- and static-pressure ratios for duct length of 30 diameters.

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